

AP Calculus BC

From the first day of class, students realize that technology and graphing calculators will be essential to their success in the course as I use the calculator to explore graphically and numerically the topic we are discussing that day. Multi-representation of functions - graphical, numerical, analytical and verbal strategies - have been used in previous courses, so most already own a TI-83, TI-84 or TI-89 calculator. Those who don't can use a TI-83 in the classroom. Students may purchase calculators at wholesale price at the beginning of the school year. Calculators are available for checking out overnight, if necessary. Each day getting the calculator out of their bookbag is just as natural as getting pencil and paper. The calculator is used frequently to help students develop a visual understanding of the concept. How to decide if the calculator is giving a correct answer is addressed early in the course. Since there are calculator and non-calculator portions on the AP test, we discuss strategies for approaching both and limit our calculator use when appropriate.

Students are given frequent opportunities to verbalize their understanding of calculus. Justification of answers is practiced early in the course and students give reasons for their answers in complete sentences throughout the year. Oral group presentations are used for exploring relationships between a function and its derivatives graphically. Each group is given the unlabeled graphs of f , f' and f'' . After discussing which graph is which function, they present their findings to the class. This is one example of how students communicate mathematically in the daily discussion. It also allows me to bring nontraditional examples into the lesson. Our textbook is excellent for analytical discussions and examples. I frequently supplement numeric and graphic situations from other sources.

Many students are taking a second year of Biology, Chemistry or Physics. We regularly discuss how calculus was used to create formulas they use in these courses. This enhances students understanding of the usefulness and importance of calculus. They share these ideas with their classmates who are not enrolled in calculus.

Unit 1: Limits and Their Properties

(11 days – one test)

- An introduction to limits, including an intuitive understanding of the limit process
- Using graphs and tables of data to determine limits
- Properties of limits
- Algebraic techniques for evaluating limits
- Special trig limits and the Squeeze theorem
- Infinite limits and one-sided limits
- Geometric understanding of the graphs of continuous functions
- Continuity by definition
- Intermediate Value Theorem

Unit 2: Differentiation

(14 days – two tests)

- Understanding of the derivative: graphically, numerically and analytically
- Approximating rates of change from graphs and tables of data using forward, backward and symmetric difference quotients
- The derivative as: the limit of the average rate of change, an instantaneous rate of change, limit of the difference quotient and the slope of a curve at a point
- Linearization of a function demonstrated by zooming-in and writing equations of tangent lines
- The relationship between differentiability and continuity
- Functions that have a vertical tangent and/or a horizontal tangent at a point
- Functions that have a point at which there is no tangent
- Rules for finding the derivative: constant, power, constant multiple, sum, difference, product, quotient
- Differentiation rules for trigonometric functions
- The chain rule

- Higher order derivatives and their meaning in the real world
- Implicit differentiation

Unit 3: Applications of the Derivative

(28 days – three tests)

- Related Rates
- Extrema on an interval and the Extreme Value Theorem
- Rolle's Theorem and the Mean Value Theorem and their geometric consequences
- Increasing and decreasing functions and the First Derivative Test
- Concavity and points of inflection and relationships to the first and second derivatives
- Second Derivative Test for extrema
- Curve sketching: use geometric and analytic information as well as calculus to predict the behavior of a function
- Explore relationships between and among the function and its first and second derivatives graphically
- Optimization including both relative and absolute extrema
- Tangent line to a curve and linear approximations
- Position, velocity, acceleration and rectilinear motion

Unit 4: Integration

(20 days – two tests)

- Slopefields and why $+C$ is so important for indefinite integrals
- Anti-differentiation, differentials and indefinite integrals
- Rules for basic integration, emphasizing that the differential of the answer gives the original integral
- Integration by substitution
- Position, velocity, acceleration and rectilinear motion
- Area under a curve
- Meaning of the definite integral
- Riemann sums and the definition of the definite integral
- Properties of the Definite Integral
- Use the Fundamental Theorem of Calculus to evaluate definite integrals
- Use of substitution of variables to evaluate definite integrals including changing limits of integration
- Average value of a function, including its graphical meaning and the Mean Value Theorem for Integrals
- Functions defined by integrals
- The Fundamental Theorem for finding derivatives of functions defined by integrals
- Estimate definite integrals using left, right and midpoint Riemann Sums and trapezoids

First Semester Exam

(three review days)

Unit 5: Transcendental Functions

(17 days – two tests)

- Derivative of logarithmic functions
- Derivative of exponential functions
- Logarithmic differentiation - when it is necessary and when it is nice
- Bases other than e and applications
- Inverse functions
- Integration involving logarithmic functions
- Integration involving exponential functions
- Inverse trig derivatives
- Inverse trig integrals

Unit 6: Differential Equations

(9 days – one test)

- Solve separable differential equations
- Find particular solutions to a differential equation given initial information
- Translate verbal descriptions of rates of change into differential equations
- Compare solutions of rates of change which are directly proportional and those that are inversely or jointly proportional
- Real-world applications including Newton's Law of Cooling and interest rates
- Use slopefields to draw solution curves for differential equations whose variables we cannot separate
- Use Euler's Method to estimate numerical solutions for differential equations whose variables we cannot separate
- Compare logistic and exponential models as solutions to differential equations.

Unit 7: Applications of the Definite Integral

(8 days – one test)

- Area of a region between curves
- Volumes of solids with known cross-sections
- Volumes of solids of revolution by disk/washer method and shell method
- Apple demonstration showing the difference between parallel and perpendicular slices
- Length of an arc
- The integral as an accumulator of rates of change
- Discussion of additional applications including work and fluid pressure

Unit 8: Techniques of Integration

(10 days – one test)

- Integration by parts
- Trigonometric integrals
- Demonstrate integration by trig substitution
- Integration by partial fractions
- Solving logistic differential equations and using them in modeling
- L'Hopital's Rule and its use in determining limits
- Identify improper integrals
- Evaluate improper integrals
- Discussion of integral tables and calculator/computer approaches to integrals which do not have an antiderivative

Unit 9: Parametric, Polar and Vector Functions

(8 days – one test)

- Show that a circle becomes a function when written in parametric form using technology
- Find first and second derivatives of parametric functions
- Slopes of parametric curves, including vertical and horizontal tangent lines
- Intersections of parametric curves
- Parametric arc length and its implication to total distance traveled
- Vectors as parametric equations
- Position, velocity and acceleration vectors
- Speed of a vector
- Polar coordinates and polar graphs
- Analysis of curves given in polar form
- Area of a region bounded by polar curves

- Explore parametric and polar mode on the calculator, noting that choices are often different from function mode

Unit 10: Infinite Series

(20 days – two tests)

- Convergence and divergence of sequences
- Definition of a series as a sequence of partial sums
- Geometric series and applications
- Find sums of infinite geometric series
- The n th-term test for divergence
- The Integral Test and its relationship to improper integrals
- Use the Integral Test to explain the test for p -series
- The Harmonic Series is a p -series with $p=1$
- Comparison tests, including Direct Comparison and Limit Comparison
- Alternating Series test and the Alternating Series Remainder
- The Ratio test and the Root test
- Discussion of polynomials and wavelets
- Taylor polynomials and approximations of functions
- Write Taylor and Maclaurin series for a given function
- Functions defined by power series
- Radius and interval of convergence of power series
- Manipulation of series: substitution, addition, multiplication, differentiation, integration and forming new series from a known series
- Taylor's Theorem with the Lagrange Form of the Remainder

Unit 11: Review for AP Test

(remaining days)

- Work free-response and multiple choice questions from previous AP tests that have not been discussed earlier in the course.
- One month in advance, students are given a project to complete. Each student is to gather information studied throughout the year and arrange it in a manner that will be useful to him/her in college and future courses. The course syllabus is used as a guide, but may be condensed or expanded as students choose. The project counts as a major grade during the sixth six weeks and is due prior to the AP exam.
- Complete the district's review and test required for second semester.

After the AP Test

- Watch the movie "Stand and Deliver"
- Study non-standard school mathematics including, but not limited to, fractals, topology and code breaking.

Primary Textbook

Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. Calculus. Houghton Mifflin Company, eighth edition, 2006. (adopted for 2007-08)

Additional Sources

AP Calculus AB & BC Practice Exams. New York: Spark Publishing, 2005.

Barton, Ray, John R. Brunsting, John J. Diehl, Greg Hill, and Karyl Tyler. Preparing for the Calculus AP Exam. Boston: Pearson Addison-Wesley, 2006.

Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic. Upper Saddle River, NJ: Pearson Education, 2003.

Foerster, Paul. Calculus Concepts and Applications. Berkeley, CA: Key Curriculum Press, 1995.

Foerster, Paul. Calculus Concepts and Applications: Instructor's Resource Book. Berkeley, CA: Key Curriculum Press, 1998.

Hughes-Hallett, Deborah, Andrew M. Gleason, and William G. McCallum. Calculus: Single Variable. Hoboken, NJ: John Wiley & Sons, Inc., 2005.

Stewart, James. Calculus: Early Transcendentals. Belmont, CA: Brooks/Cole- Thomson Learning, 2003.

The 2003 AP Calculus AB and AP Calculus BC Released Exams. New York: College Entrance Examination Board, 2005.